Synchronization and Intermittency of Type I in the Oscillator Model of Heart Rhythm

N.T. Abdullayev*, O.A. Dyshin and D.A. Dadashova

Azerbaijan Technical University, Department of Television and Radio Systems, Baku, Azerbaijan

Abstract: With the help of the truncated equation investigated the intermittent behavior of the oscillator Van der Pol oscillator under periodic external action in the absence and in the presence of noise. A method for determining when reinjection by constructing a singular component of the power spectrum of the signal by the flicker noise spectroscopy.

A procedure for determining the current parameters of the oscillator based on the calculation of the wavelet coefficients of the signal system using fast discrete wavelet transform and application of the differentiation formulas wavelet expansions.

Keywords: Oscillator, range of power, flicker-noise spectroscopy, veyvlet-transform, area of synchronization.

One known scenario of transition to chaos is the transition through intermittency. Under intermittency mean [1], this kind of signal, which randomly alternate long regular (laminar) phase (so called window) and relatively short irregular bursts. This state of the system, described by the data signal, commonly referred to as an attractor type noisy cycles [2]. It is noticed that the random number of bursts increases with increasing external parameter, which means that intermittency is a continuous transition from regular to chaotic motion. The statistical characteristics are studied for a long time, in connection with the distinction between the different types of intermittency, such as the intermittency of type I-III [1,3], on-off intermittency [4-5], intermittency "needle's eye" [6-7] and intermittence cycle [8].

According to the conceptual framework of flicker noise spectroscopy [9], the most common to view the evolution of the dynamic variable for th space-time level is presented in the form of intermittency, when not all the slots on the time axis information equivalent. This pattern is characterized by relatively weak changes in a variable on a relatively extended time intervals - "laminar phases" with typical durations T_0^i and abrupt interruptions of such abrupt changes of evolution of the value of a dynamic variable in the short duration of the intervals τ_0^i ($\tau_0^i \leq T_0^i$). Due to the inertia of the system, each such an abrupt change in the dynamic variable values may be associated with dynamic bursts - and a sharp increase in short-term relaxation $V_i(t)$ with attenuation values on the subsequent "laminar" plot, leading to rupture of derivatives related to "laminar" sections. The magnitude and duration of these bursts are specific to each system, causing a certain contribution to the corresponding power spectrum. Such jumps and discontinuities are in [9] for the first jumps and breaks derivatives type, assuming that the variable $V_i(t)$ can be characterized by sharp changes in the intervals of intermittent "laminar" background. For the typical time intervals between such sharp jumps (they are referred to jumps of the second type) is introduced designation believing. It is believed that all the basic information about the evolutionary process for th hierarchy level contains only these bursts, jumps and discontinuities input signal. All these irregularities are considered as the main and the only "tokens" of the evolutionary process.

There is no doubt that the various types of intermittent behavior can occur in a wide range of systems, including self-oscillating system, in the presence of noise and fluctuations. In this paper, an analysis of the transition to synchronization modes (frequency capture, phase, phase jumps) in the oscillator Van der Pol at periodic external action in the absence and in the presence of noise. With the help of the truncated equation to investigate the process of destruction of synchronization with a small phase mismatch of foreign forces and their own self-oscillation in the event of a local saddle-node bifurcation on the boundary of the synchronization (often called the recent Arnold language). In close proximity to the language border observed intermittency of type I: extensive areas of almost constant phase ("laminar" phase) are separated by relatively short sections of "slippage" phase, where it is changed to a value close to 2π ("turbulent" phase). This noise makes the main features in the intermittent behavior when the system in the absence of type I intermittency shows.

When reinjecting produced damping vibration process. Moment of reinjection, leading to relaminarization determined by us by constructing a singular component of the power spectrum of a signal generated by the system.

*Address correspondence to this author at the Azerbaijan Technical University, Department of Television and Radio Systems, Baku, Azerbaijan;
E-mail: a.namik46@mail.ru

E-ISSN: 2311-052X/16

In contrast to the existing methods for determining the current values of the parameters of the nonlinear oscillator, in this paper, we propose an alternative version of the technical detection procedure, based on the calculation of the analyzed system signal wavelet coefficients using fast wavelet transform and application of the formulas of differentiation of the discrete wavelet decomposition, which reduces the problem of the definition of oscillator options to solving a system of linear algebraic equations in the space of wavelet coefficients. This approach not only provides a significant gain in computational speed by the use of fast (pyramidal) algorithm of discrete wavelet transform, but also a significantly higher noise immunity compared to the procedure of using the approximate formulas of numerical differentiation, based on Newton's interpolation polynomials.

1. GO TO THE SYNCHRONIZATION AND INTERMITTENCY OF TYPE I IN THE OSCILLATOR VAN DER POL AT PERIODIC EXTERNAL INFLUENCE DURING THE ABSENCE OF NOISE

Consider the nonautonomous generator Van der Pol oscillator under external harmonic action.

$$\ddot{x}-\left(\lambda-x^2\right)\cdot\dot{x}+x=B\cdot\sin\left(\omega t\right). \tag{1.1}$$

Here, the dimensionless parameter determines the amplitude, and ω- exposure frequency. If the system is described by equation (1.1) is close to the threshold of self-oscillations (λ small), the amplitude of these oscillations and the amplitude of the effects are small, and the frequency of exposure close to the frequency of small natural vibrations, you can use a variant of the method of slowly varying amplitudes, for example, method of complex amplitudes proposed by Van der Pol [10,11], according to which the solution of the equation (1.1) is sought in the form of a quasi-harmonic oscillation with slowly varying amplitude A(t)

$$x\left(t\right)=\mathrm{Re}\left(A\left(t\right)e^{i\omega t}\right). \tag{1.2}$$

Substituting (1.2) (1.1) by averaging over the period, to arrive at truncated equation [11]

$$\dot{A}+i\frac{\left(\omega^2-1\right)}{2\omega}A=\frac{\lambda A}{2}-\frac{|A|^2\cdot A}{8}-\frac{B}{2\omega}\ . \tag{1.3}$$

We are interested in the case when the autonomous system performs self-excited oscillations, ie, λ>0 . Believing

$$\tau=\frac{\lambda t}{2},\ z=\frac{A}{\sqrt{4\lambda}},\ \Delta=\frac{\left(\omega^2-1\right)}{\lambda\omega},\ \eta=\frac{B}{2\omega\lambda^{3/2}}\ , \tag{1.4}$$

z(τ) rewrite (1.3) in the unknown function:

$$\dot{z}+i\Delta z=z-|z|^2\cdot z-\varepsilon\ , \tag{1.5}$$

now where the dot denotes differentiation with respect to the parameter is the renormalized amplitude of external influence Δ- influence on the frequency detuning of the natural frequency of oscillation ω_0.

The parameter η is defined in (1.4) through the attitude *B* and $\lambda^{3/2}$ values that are applicable to the method of slowly varying amplitudes of the two should be small. However, the relationship between them can be set in any way, only the parameter was small. This is ensured, for example $B<<\lambda^{3/2.}$. The applicability of this approach is, therefore, a double inequality $B<<\lambda^{3/2}<<1$

Let's present complex amplitude in the form $z=\mathrm{Re}^{i\phi}$. Then from the equation (1.5) we receive.

$$\dot{R}e^{i\varphi}+iR\dot{\phi}e^{i\varphi}+i\Delta Re^{i\varphi}=Re^{i\varphi}-R^3e^{i\varphi}-\eta\ . \tag{1.6}$$

Multiplying this equation $e^{i\phi}$ by and separating the real and imaginary parts, we obtain

$$\begin{aligned}\dot{R}&=R-R^3-\eta\cdot\cos\phi\\ \dot{\phi}&=-\Delta+\left(\eta/R\right)\sin\phi.\end{aligned} \tag{1.7}$$

Assume that the amplitude of the impact is small, i.e. parameter is small. In the zero-order in η the first equation (1.7) we find the amplitude of the stationary vibrations $R=1$ and substitute it in the second equation. As in the second equation the corresponding term contains a ε factor, when this substitution can be used for *R* the zero-order approximation. As a result, we obtain a closed equation for a single variable - in relation to the external action phase oscillations of the system.

$$\dot{\phi}=\Delta+\eta\cdot\sin\phi\ . \tag{1.8}$$

In the foreign literature it is called the equation of Adler.

We introduce the function

$$U\left(\phi\right)=\phi\cdot\Delta+\eta\cdot\cos\phi=\Delta\left[\phi+\left(\eta/\Delta\right)\cos\phi\right] \tag{1.9}$$

and write the equation (1.8) in the form

$$\dot{\phi}=-\partial U/\partial\phi\ . \tag{1.10}$$

From the point of view of the form of the potential function (1.9), significant is its dependence on the relationship η/Δ. Since $\frac{\partial U}{\partial\phi}=\Delta\left[1-\left(\eta/\Delta\right)\sin\phi\right]$, when the $|\eta/\Delta|<1$ potential depends monotonically ϕ, as in

$1-(\eta/\Delta)\sin\phi > 0$ this case coincides with the sign Δ. When $|\eta/\Delta| = 1$ another phase dependence is monotonic $1-(\eta/\Delta)\sin\phi \geq 0$ as but acquires the inflection point (if $\frac{\partial^2 U}{\partial\phi^2} = 1-(\eta/\Delta)cos\phi = 0$) where the tangent is horizontal (the point at which the simultaneous $\frac{\partial U}{\partial\phi} = 0$ and $\frac{\partial^2 U}{\partial\phi^2} = 0$ equations satisfy relations, $\sin\phi = \cos\phi = \frac{1}{\varepsilon/\Delta}$ i $\eta/\Delta = 1$ when and $\eta/\Delta = -1$ under) , and the graph of the function in this case has the form of locally cubic parabola $y = x^3$ (critical situation). Finally, $|\eta/\Delta| > 1$ when the potential function $U(\phi)$ has highs and lows, as changes its $\frac{\partial U}{\partial\phi}$ sign at some ϕ. Figure **1** [11] shows the area where is realized the first and third situations in the parameter plane (Δ,η). The critical situation occurs on the delimiting lines $\Delta = \pm\varepsilon$.

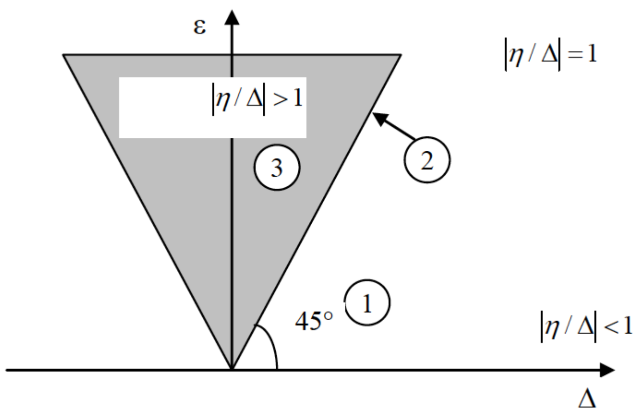


Figure 1: Field of sync or Arnold language (gray) on the plane (Δ,η).

To study the phenomenon of synchronization will assume that self-oscillation quasi-linear, ie, solution of

$$\ddot{x} - (\lambda - x^2)\dot{x} + x = 0 \quad (1.11)$$

It represented as

$$x(t) = A_0 \sin(\omega_0 t + \phi_0)$$

a frequency ω_0 and amplitude A_0, and an external force - is harmonic with the frequency ω, i.e. force has the form $B\cos(\omega t + \overline{\phi_e})$ where - power phase ($\overline{\phi_e}$ phase be the initial force, and - its amplitude is important that the frequency of power is generally different from the frequency of oscillations ω_0, which is called the autonomous rate; $\omega - \omega_0$ detuning difference is called [10].

In the reference system rotating with angular velocity ω, force $B\cos(\omega t + \overline{\phi_e})$, oscillating at a frequency granted permanent vector B of length acting at an angle $\phi^\circ = \overline{\phi_e} + \pi/2$ of quasi-linear self-oscillations. The equilibrium position ϕ° is asymptotically stable, while the phase ϕ_0 of the oscillation is unstable asymptotically independent and stable only neutral.

Let us first consider the simplest case of zero detuning, ie, when $\omega = \omega_0$. Thus, regardless of the initial phase difference $\phi - \phi_e$, the phase point moves towards a stable equilibrium, so in the end $\phi = \phi_e + \phi^\circ$, ie phase of oscillation force captured.

Of course, the synchronous mode is set for a very long time. This case is quite trivial since, by assumption, frequency ω and ω_0 coincide with the start, so that synchronization occurs only to establish a certain ratio between the phases.

Suppose now that the frequency of power is different ω from the frequency of autonomous oscillations ω_0, we assume for definiteness $\omega_0 > \omega$. Consider the impact of external forces on the slowly rotating phase point. At a certain phase difference value of the rotation $\Delta\phi = \phi - \phi_e - \phi^\circ$ force balances and stops him. As a result, self-oscillation frequency of the driving Ω, called the observed frequency becomes equal to the frequency power $\Omega = \omega$, and sustainable relationship is established between the phases. Such a motion is called synchronized [10]. If synchronization is not the same phase, but their difference is constant $\phi - \phi_e = \phi^\circ + \Delta\phi$; Angle $\Delta\phi$ called phase shift. For sufficiently small frequency detuning Ω is captured - it becomes ω. If the deviation exceeds a certain threshold, then the equality $\Omega = \omega$ is violated. Coincidence frequency within a finite range of detuning is the basic sync feature, called the seizure frequency.

If the deviation exceeds a certain threshold, the force can not stop the rotation. With increasing mismatch (i.e., with increasing speed of rotation $\omega_0 - \omega$) state, the equilibrium is shifted towards the point where the maximum force retarding action. In the end, the stable and unstable equilibrium state collide and disappear, and the phase point starts to rotate with the so-called beat frequency Ω_b. Although this mismatch external power is not sufficient for synchronization, it significantly affects the dynamics: the power of making the rotation uneven and the average protects it, so $\Omega_b < \omega_0 - \omega$.

Returning to the original frame of reference we see that the self-oscillation frequency $\omega+\Omega_b<\omega_0$ and have a growth phase modulated at a frequency of heartbeat Ω_b. This movement is characterized by two frequencies ($\omega+\Omega_b$ and Ω_b) are called quasi-periodic. More specifically, the movement - quasiperiodic-cal, if the ratio $(\omega+\Omega_b)/\Omega_b$ is irrational, that is a typical situation.

Thus, even a small force can lead to the seizure frequency of oscillation. The greater the mismatch, the greater force is required for synchronization.

The family of curves depending $\Omega-\omega$ on the ω force for different values of the amplitude possible B to determine the area of a plane (ω,B) **,** corresponding to the synchronized oscillator condition (Figure **2** [10]), ie, the area in which the frequency of the driving Ω oscillation frequency is equal to the outside ω (Figure **2**, it is shaded).

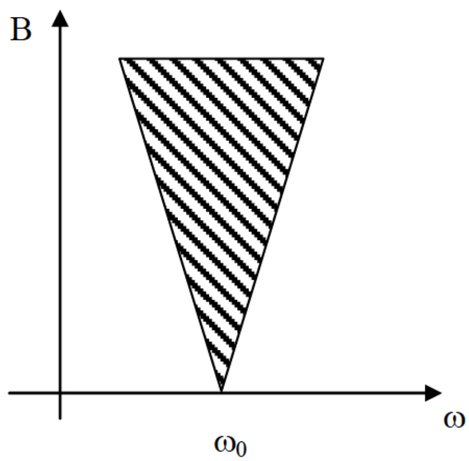


Figure 2: Dependence $\Omega-\omega$ on ω for different values of the amplitude of the force B.

This area is called domain synchronization or tongue Arnold. At low limits of language - direct line: it is the general case for weakly perturbed self-oscillations (Figure **3** [11]).

For large form of the language depends on the specific properties of the self-oscillation and power. Area synchronization concerns axis. This means that at zero detuning self-oscillation can be synchronized with an arbitrarily small force (although in this case, the transient state may be synchronized to last indefinitely), so that powerful oscillator frequency can be stabilized by weak but high-quality of the auxiliary signal generator.

Synchronization is often described in terms of the capture phase. In asynchronous movement occurs unlimited growth phase difference, while in the synchronous mode the phase difference is limited and there is a constant phase shift between the oscillations and strength.

$$\phi(t)-\phi_e(t)=const\ , \tag{1.12}$$

where the constant is $\phi^{\circ}+\Delta\phi$. Required for phase locking to the phase difference remained limited in a finite region detuning i.e. inside the synchronization region.

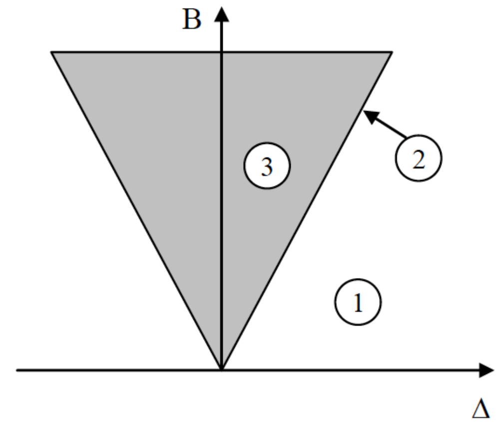


Figure 3: Scope or synchronization Arnold Language (gray) on a plane (Δ,B), where - Δ frequency detuning, B - the amplitude of the impact; 3 - Sync area $(B/\Delta)>1$, 2 - the boundary of the synchronization region $(\Delta=\pm B)$.

Choose ω in the middle of the field synchronization and consider how synchronization is lost when crossing language boundaries due to changes in the frequency of exposure. Let's start with zero development, $\omega_0=\omega$ and will be gradually reduce the frequency of the external force ω. If the deviation is zero, then the phase difference $\Delta\phi=\phi^{\circ}$; For definiteness, we assume. With increasing detuning becomes non-zero phase shift. When the point crosses the language border, there is a loss of synchronization and the phase difference increases indefinitely. However, this growth is not uniform: there are periods of time when the phase difference is almost constant, and shorter periods, when the phase difference rather quickly increases by 2π. This rapid change, looks like a leap, called the slip phase (phase slip). phase jump speed depends on the amplitude of the force. With a weak impact of breakthrough lasts a few may even force many periods.

Synchronous and slip alternate intervals, so that we can say that the dynamics of the phase difference - intermittent. With a further increase in the duration mismatch almost synchronous intervals becoming smaller, and eventually increase the phase difference becomes almost uniform. [10]

Character mode system (1.1) outside the synchronization region (when $|\Delta| > \eta$) it is easy to install with the help of the differential equation (1.8), which in this case has an analytical solution [11]:

$$\phi = \frac{\pi}{2} - 2\,arctg\left[\sqrt{\frac{\Delta-\eta}{\Delta+\eta}}\,tg\left(\frac{1}{2}\sqrt{\Delta^2-\eta^2}\,\tau\right)\right]. \tag{1.13}$$

Let's expression (1.13) the dependence of the phase on time is given by the period $T = 2\pi\sqrt{\Delta^2-\eta^2}$ of oscillations superimposed on a linear drift with an average speed of phase (Figure **4** [11]).

$$\langle\dot{\phi}\rangle = \begin{cases} -\sqrt{\Delta^2-\eta^2}\,, & \Delta > 0, \\ \sqrt{\Delta^2-\eta^2}\,, & \Delta < 0. \end{cases} \tag{1.14}$$

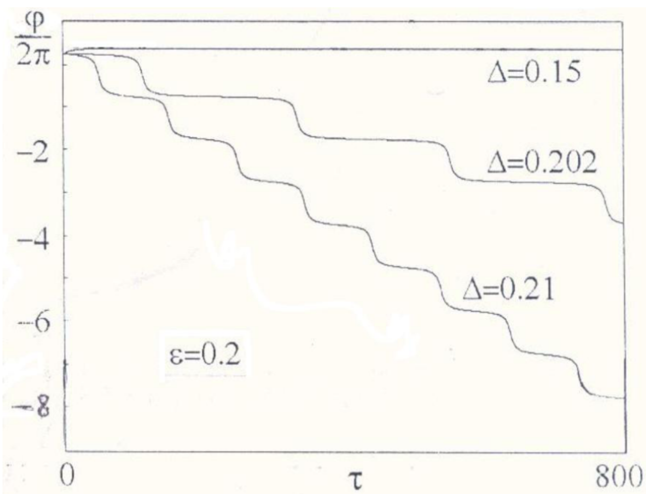


Figure 4: The time dependence of the phase at ε=0.2 in synchronization tongue (Δ=0.15) and the tongue is in close proximity to the border ((Δ=0.202) and a somewhat greater distance from it ((Δ=0.21).

From Figure **4** shows that in the vicinity of the boundary can be observed tongue portions almost constant phase separated by relatively short sections "slip" phase, where it is changed to a value close. Phase transformation depending on approaching the boundary of the synchronization area is to increase the permanence phase plots.

For small amplitudes of the first exposure to the approximate solution of equation (1.7) is the $R=1$ solution of the truncated equation (1.5) is represented in the form $z = e^{i\phi}$. Then the solution of equation (1.1)

$$x = \mathrm{Re}\,Ae^{i\omega t} = 2\sqrt{\lambda}\,\mathrm{Re}\,ze^{2i\omega\tau/\lambda} =$$

$$2\sqrt{\lambda}\left[\cos\phi\cdot\cos\left(\frac{2\omega\tau}{\lambda}\right) - \sin\phi\cdot\sin\left(\frac{2\omega\tau}{\lambda}\right)\right] =$$

$$=2\sqrt{\lambda}\,\frac{\sqrt{\Delta^2-\eta^2}\,\sin\left(\sqrt{\Delta^2-\eta^2}\,\tau\right)\cdot\cos\left(\frac{2\omega\tau}{\lambda}\right) - \left(\Delta\cos\left(\sqrt{\Delta^2-\eta^2}\,\tau\right)+\eta\right)\cdot\sin\left(\frac{2\omega\tau}{\lambda}\right)}{\Delta+\eta\cos\left(\sqrt{\Delta^2-\varepsilon^2}\,\tau\right)}.$$

The process described by expression (1.15) comprises two frequency corresponding to the fast and slow fluctuations, respectively,

$$\Omega_0 = 2\omega/\lambda \text{ и } \Omega_1 = \sqrt{\Delta^2-\eta^2}\,. \tag{1.16}$$

They may be in an arbitrary, including irrational regard. Then we are not dealing with the periodic and quasi-periodic oscillation with the process. This oscillatory regime is realized outside the scope of synchronization is called beat regime with the beat frequency $\Omega_b = \sqrt{\Delta^2-\eta^2}$ [12].

Let us see what happens in the phase plane $(x = \mathrm{Re}\,z,\, y = \mathrm{Im}\,z)$ of the truncated equation (1.5), when we pass the boundary of the synchronization $(|\Delta| = \eta)$ at low amplitude effects. For small Δ and large η stable fixed point $z^* = const$ (singular point - node) in the complex plane corresponds to the amplitude of the synchronous mode, and the destruction of local synchronization corresponds to a saddle-node bifurcation associated with the global bifurcation of a limit cycle from a separatrix of the saddle and the general assembly [13,14]. Below boundary timing mode (for small frequency detuning Δ) in the phase difference versus time

$$\Delta\phi(t) = \phi(t) - \omega t \tag{1.17}$$

(where $\phi(t)$ - the phase of the Van der Pol oscillator) contains lots of synchronous dynamics (laminar phase), discontinuous phase slip (turbulent phase), during which the value $|\Delta\phi(t)|$ is changed to 2π .

The average duration of laminar phases T is dependent on the critical parameter setting according $\varepsilon = (B_c - B)$ to the law

$T \sim (\varepsilon-\varepsilon_c)^{-1/2}$, (1.18) corresponding intermittency of type I. The critical amplitude of the external force B_c, resulting in the destruction of synchronization due to a saddle-node bifurcation, is calculated from the relationship

$$\Delta = \eta \Rightarrow \frac{\omega^2-1}{\lambda\omega} = \frac{B}{2\omega\lambda^{3/2}} \Rightarrow B_c = 2\pi\cdot\left(\omega^2-1\right). \tag{1.19}$$

The classic model for the study of type I intermittency is [15] a one-parameter quadratic map

$$x_{n+1} = f(x_n) = x_n + x_n^2 + \varepsilon \ , \qquad (1.20)$$

where ε - the control parameter. The value $\varepsilon_c = 0$ corresponds to a saddle-node (tangent) bifurcation, where stable and unstable fixed points $x_{u,s} = \pm|\varepsilon|^{1/2}$, merge with each other at a point and disappear.

Below the critical value of the parameter $\varepsilon < \varepsilon_c$ there is a stable fixed point $x_s = -|\varepsilon|^{1/2}$, while above there is ε_c a narrow corridor between the graph of the function $f(x)$ and the $x_{n+1} = x_n$ bisector, so that the image point of the mapping (1.20) moves along it (Figure **5** [16]).

This movement corresponds to a laminar phase, and its average duration T is inversely proportional to the square root of, $(\varepsilon - \varepsilon_c)$, the asymptotic equality (1.18) [17].

2. SYNC AND INTERMITTENCY OF TYPE I IN THE OSCILLATOR VAN DER POL AT PERIODIC EXTERNAL ACTION IN THE PRESENCE OF NOISE

Consider the nonautonomous generator Van der Pol

$$\ddot{x} - (\lambda - x^2)\dot{x} + x = b\sin(\omega - t) + D\xi(t) \qquad (2.1)$$

under the influence of an external harmonic with amplitude b and frequency ω, while the presence of a stochastic term which $D\xi(t)$ - delta-correlated rovanny white noise $[\langle \xi(t)\rangle = 0, \langle \xi(t)\xi(\tau)\rangle = \delta(t-\tau)]$. Here $\langle\cdot\rangle$ - averaging sign $\delta(t)$ - Dirac function.

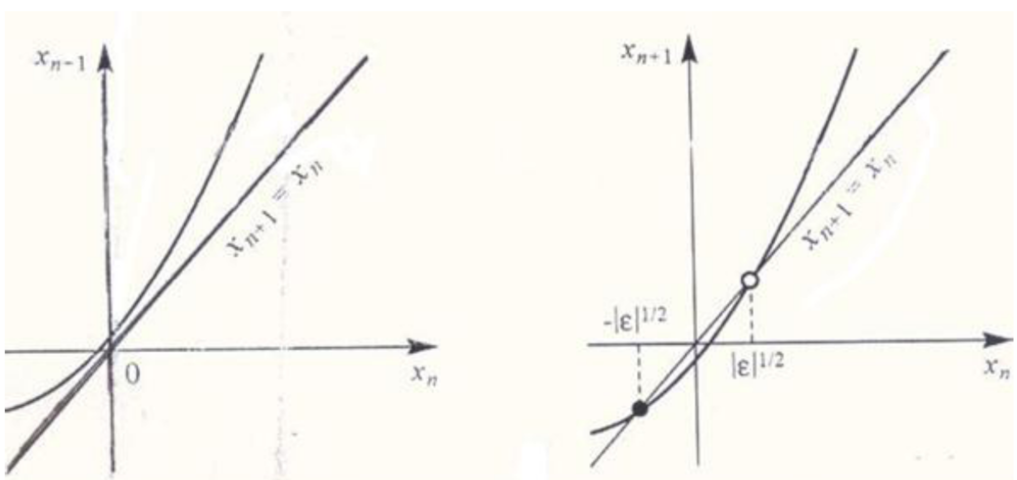
When considering the synchronization of self-oscillations of noise are distinguished [10] for the case: (i) a faint noise limited and (ii) unlimited or limited strong noise.

Weak limited noise at low frequency deviation Δ does not result in leakage, noise can not throw a particle from one building to another minimum. In this case, $\Delta\phi$ the phase difference fluctuates in the presence of random noise, but remains limited, and the condition of phase locking

$$|\phi_e - \phi| < const \qquad (2.2)$$

it is still valid. The average frequency Ω of oscillation and noise captured by force. In larger mismatch noise intensity becomes sufficient to overcome the potential barrier, and the particle begins to slide down the significance of the graph, wherein the transition occurs at a lower detuning, than in the absence of noise.

If the noise is unlimited (ie, Gaussian), or a limited, but powerful, it skips phase. When you angle the potential jumps down occur more frequently, and the average phase difference grows at an arbitrarily weak detuning. Strictly speaking, unlimited noise destroys synchronization, so that neither the capture phase condition (2.2), or the condition of frequency capture $\Omega = \omega$ formulated for the deterministic case, are not fulfilled. Nevertheless, at least for low noise, one can speak of an approximate equal frequency in a range of detuning. With this range increase of noise intensity



a) b)

Figure 5: Iterative diagram display for ε>0 (a), and ε<0 (b). Stable $x_s = |\varepsilon|^{1/2}$ and unstable $x_u = |\varepsilon|^{1/2}$ fixed points of the map are marked • and ^{0} respectively.

decreases, and the synchronization is shown as a weak "pull-up" of the observed frequency to the frequency of the external force ω.

For noise systems, generally speaking, it is impossible to talk about the capture phase, because the phase difference is not limited. On the other hand, the particle is often a minimum capacity, and therefore a certain $\phi - \phi_e$ value is more likely to occur. The existence of a preferred phase difference values $\phi - \phi_e$ can be interpreted as a statistical analog phase locking. A similar distribution of phases is observed in the self-oscillations without noise, near the transition from the phase synchronization when the speaker - intermittent (see Figure **3**).

To construct a theory of intermittency of type I in the presence of noise [16] the quadratic mapping (1.20) with the addition of a stochastic term ξ_n:

$$x_{n+1} = x_n + x_n^2 + \varepsilon + \xi_n \, , \tag{2.3}$$

where ξ_n - delta-correlated white noise with zero mean $\left[\langle \xi_n \rangle = 0, \langle \xi_n \xi_m \rangle = D\delta(n-m)\right]$.

The degree of influence of the stochastic ξ_n term behavior of the system is governed by the value of the parameter D (noise intensity). For positive values of the parameter $\varepsilon \left(\varepsilon > 0\right)$, the image point on the chart corresponding to the iterative behavior of the system (2.3), moves along a narrow corridor, and this motion is perturbed randomly. While the intensity of the noise is small, there are characteristics that are close to those of the classical type of intermittency I, as described at the end of section 1.

A different scenario occurs if the control parameter ε takes negative values ($\varepsilon < 0$). In this case, the point corresponding to the behavior of the system (2.3), for a long time remained in the area $x < x_c = |\varepsilon|^{1/2}$, and the dynamics of the system are also outraged by the random force. Once the image point due to the influence of the noise reaches the limit $x_c = |\varepsilon|^{1/2}$, the system begins the turbulent phase, but such an event occurs infrequently.

In this case, display behavior (2.3) is fundamentally different from the dynamics of the system (1.20), since (1.20) is the turbulent phase is not observed in the absence $\varepsilon < 0$ of noise in. Consequently, the negative values of the parameter ε is an important concern when considering the intermittency of type I in the presence of noise

Assuming that the ε value is negative and small, and the x value is changed in one iteration slight, can be seen the difference $\left(x_{n+1} - x_n\right)$ of a derivative with respect to time and switch from a system with discrete time (2.3) to a streaming system, a continuous-time described by stochastic differential equation similar to that carried out in the classical theory of intermittency of type I. The resulting stochastic differential equation is equivalent to the Fokker-Planck equation, using that in [16] it is shown that the distribution of the duration of laminar phases is expressed by the exponential law

$$p(t) = T^{-1} \exp(-t/T) \, , \tag{2.4}$$

Where T - the average duration of laminar phases, defined by the expressio

$$T = \frac{1}{k\sqrt{|\varepsilon|}} \cdot \exp\left(\frac{4|\varepsilon|^{3/2}}{3D}\right), \tag{2.5}$$

k -proportionality factor

The analytical expression (2.5) for an average duration is consistent with the decision of the laminar phases

$$T \sim |\varepsilon|^{-1/2} f\left(\sigma^2 |\varepsilon|^{-3/2}\right), \tag{2.6}$$

derived in previous studies [18-20] and coincides with the formula for, given in [21]. Another critical parameter value of the parameter ε is large enough, you can use the approximate equation [21].

$$\ln T \sim D^{-1} |\varepsilon|^{3/2} \, . \tag{2.7}$$

Previously intermittent behavior in the presence of noise has been studied in [18] using the Fokker-Planck equation on the basis of renormalization-group analysis, but the characteristic patterns were found only in the subcritical region where the intermittent behavior is observed both in the presence of noise and when no. In the supercritical region $\left(\varepsilon < 0\right)$, where there is no noise intermittency does not arise, theoretical consideration Move-ment behavior of the presence of noise was carried out in [21]. At the same analytical form according to the average duration of laminar phases of supercritical parameter ε was obtained under the assumption of a fixed-term probability reinjection external action taken in the form δ- function. At the same time, another important statistical characteristics of intermittent behavior, namely the distribution of the duration of laminar phases is not considered in the supercritical region of parameters of control values.

In [16] analytically derived form of the distribution of the laminar phases for the intermittency of type I in the presence of noise in the supercritical region of the control parameter values. It therefore follows already known dependence of the average duration of laminar phases of the critical parameter setting [21,22]. Moreover, it was shown that the pattern obtained in [21], assuming a fixed probability reinjection selected as δ-function, actually, almost independent of the nature of the process and relaminarization respectively, the resulting expression for the average duration of the phase parameter laminar supercritical remains valid in various forms of probability reinjection.

However, proposed in [16] The theory explains the emergence of a long laminar regions and does not say anything about the description of chaotic bursts that require additional research. Total phenomenological (nemodelny) approach to the analysis of irregularities - bursts based on the flicker-noise spectroscopy, proposed in [9] and is used in Section 3 for the construction of a singular component of the power spectrum of a signal.

It is also one of the most interesting phenomena in coupled self-oscillatory systems is the effect of vibrations of the death (amplitude death, oscillator death). This effect is that the dissipative coupled avtokole-vibrational oscillators with different natural frequencies with the withdrawn-cheniem communication occurs damping of oscillation and transition to steady state exposure.

To restore the self-oscillation pulse is used reinjection periodic influence on the oscillatory system. Thus, in the pacemaker models just used a peripheral signal to correct an abnormally slow heart rate (bradycardia). In such devices, the electrical impulses from the implanted generator stimulates the driver of cardiac rhythm (pacemaker) first order - the sinoatrial node - and cause the contraction of the heart. Modern pacemakers - a programmable adaptive devices that use complex algorithms to monitor cardiac activity; they change in heart rate depending on time of day, the patient's physical activity, etc.

Moment of reinjection may be determined from the power spectrum analysis of one-dimensional dynamic variable that describes $x(t)$ the behavior of the system (2.1), which is described in detail in Section 3.

3. CONSTRUCTION OF A SINGULAR COMPONENT OF THE POWER SPECTRUM OF THE SIGNAL

In order to assess the type of irregularities bursts will use the method of flicker-noise spectroscopy (FNS), by which the estimated parameters of the singular component of the power spectrum of the signal. The method of flicker noise spectroscopy is a common phenomenological (nemodelnym) approach to the analysis of chaotic signals of different nature [9]. The essence of the FNS approach is to give information of significance of correlations, which are implemented in the signal sequences of irregularities - bursts, breaks of the derivatives of various orders - as a media measurement information, occurring at every level of spatio-temporal hierarchical organization of the study of the dynamical system.

For a one-dimensional signal $V(t)$ defined in the interval $[0,T]$ from discrete steps Δt at points $t_k = k\cdot\Delta t (k=1,\ldots,N)$, $N=[T/\Delta t]$, its mean value is calculated $\langle V(t)\rangle = \frac{1}{N}\sum_{k=1}^{N} V(t_k)$. We believe further that a signal $V(t)$ stationary and $\langle V(t)\rangle = 0$. The spectrum of the signal $S(f)$ power is defined as the

Fourier transform of the autocorrelation function. When calculating the autocorrelation function

$$\psi(\tau) = \langle V(t)V(t+\tau)\rangle \tag{3.1}$$

we will assume

$$f \le f^* \left(f^* = f_{\max}, f_{\max} = 1/\Delta t\right),\ \tau \le \tau^* \left(\tau^* = T/4\right), \tag{3.2}$$

where f - frequency τ - time delay setting.

For a stationary signal $S(f) = S_c(f)$ where $S_c(f)$ - cosine transform, the Fourier of the autocorrelation function.

In the discrete case "experimental" power $S(f)$ spectrum calculated by the trapezoid method:

$$S_c(f) = \frac{1}{\Delta t} S_c(q) \tag{3.3}$$

$$\begin{aligned} S_c(q) &= \psi(0) + \psi\left(\frac{M}{2}\right)(-1)^q \\ &+2\sum_{m=1}^{M/2-1} \psi(m)\cdot\cos\left(\frac{2\pi q m}{M}\right)\ (q = 0,1,\ldots,M-1) \end{aligned}, \tag{3.4}$$

$$q = f\cdot T_M,\ T_M < T,\ T_M = M\cdot\frac{T}{N} = M\cdot\Delta t\ , \tag{3.5}$$

$$f = \frac{q}{T_M} = \frac{q}{M\cdot\Delta t} = \frac{q/M}{\Delta t}\ , \tag{3.6}$$

$M = \left[\frac{T_M}{T}\cdot N\right]$ an even number of points on the frequency axis.

For any $m = [\tau / \Delta t]$ и $N = [T / \Delta t]$ "experimental" autocorrelator is calculated according to the formula

$$\psi(m_\tau) = \frac{1}{N - m_\tau} \sum_{k=1}^{N-m_\tau} V(k) V(k + m_\tau) \quad (m_\tau = 0,1,\ldots,M-1). \tag{3.7}$$

The number M should satisfy the relation

$$\frac{4}{3} \le M \le N. \tag{3.8}$$

The power $S(f)$ spectrum has a negative even function f with a period of 2π [23].

Let $V_R(t)$ and $V_F(t)$, respectively, low and high frequency components of the signal. Isolation of high-frequency component based "relaxation" procedure [24] built on the analogy with the solution of the diffusion equation with the diffusion coefficient χ

$$\frac{\partial V}{\partial \tau} = \chi \cdot \frac{\partial^2 V}{\partial t^2}, \tag{3.9}$$

It is represented as a difference equation

$$\frac{V_k^{j+1} - V_k^j}{\Delta \tau} = \chi \cdot \frac{V_{k+1}^j + V_{k-1}^j - 2V_k^j}{(\Delta t)^2}, \tag{3.10}$$

corresponds to a simple difference scheme for numerical solutions of differential equations.

From (3.10) we obtain

$$\mathrm{V}_\mathrm{k}^{\mathrm{j}+1} = \mathrm{V}_\mathrm{k}^\mathrm{j} + \frac{\chi \cdot \Delta \tau}{(\Delta \mathrm{t})^2} \cdot \left(\mathrm{V}_{\mathrm{k}+1}^\mathrm{j} + \mathrm{V}_{\mathrm{k}-1}^\mathrm{j} - 2\mathrm{V}_\mathrm{k}^\mathrm{j}\right). \tag{3.11}$$

Introducing the notation $\omega = \frac{\chi \cdot \Delta \tau}{(\Delta t)^2}$, we rewrite the last equation in the form

$$V_k^{j+1} = \omega V_{k+1}^j + \omega V_{k-1}^j + (1 - 2\omega) V_k^j. \tag{3.12}$$

From the theory of the stability of difference schemes [25] it is known that the difference scheme is absolutely stable at $\omega < 1/2$. To use equation (3.12) in the boundary conditions must be set as the smoothing procedure. In this case, these terms are defined as follows.

Let smoothing is carried out for a series of dots in M length. Then, at each step of the extreme values of smoothing iterations $k = 1$ and $k = M$ are calculated according to the formulas

$$V_1^{j+1} = (1 - 2\omega) V_1^j + 2\omega V_2^j, \; V_M^{j+1} = (1 - 2\omega) V_M^j + 2\omega V_{M-1}^j. \tag{3.13}$$

Iteration of these equations, i.e. calculation on th new "relaxation" of the signal V_k^{j+1} values $(j+1)$-of the values of step V_k^j (taken when the signal $V(t)$) itself) provides a low-frequency component V_R. The high-frequency component is obtained as the difference $V_F = V - V_R$.

Construct the image $S(f)$ dvulogarifmicheskoy scale ($\log S(f), \log f$) (Figure **6** [9]).

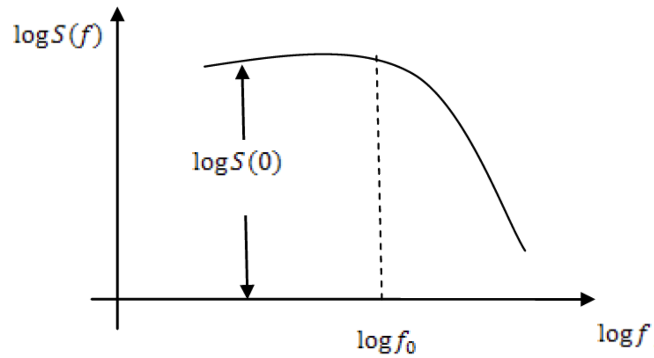


Figure 6: The asymptotic behavior of the power spectrum $S(f)$ at high and low frequencies.

If for some f have $S(f) < 0$, instead $S(f)$ of being considered $|S(f)|$.

We have the following asymptotic representation

$$S(f) \to \begin{cases} 1/f^n, \; ec\pi u \, f >> f_0, \\ S(0), \quad ec\pi u \, f << f_0. \end{cases} \tag{3.14}$$

Here

$$f_0 = 1/(2\pi - T_0), \tag{3.15}$$

T_0- defines a characteristic time within which realized the relationship of the measured dynamic variable $V(t)$; dimensionless parameter n effectively determines the way in which this relationship is lost as to reduce the frequency values f_0.

From Figure **6** that is f_0 approximately equal f_0^*, to the frequency from which ceases to stabilize $S(f)$ around a certain constant $S(0)$.

The singular component $S_s(f)$ of an experimental power spectrum $S(f)$ is calculated according to the formulas (3.3) - (3.4) with replacement $\psi(t)$ autocorrelator, calculated by the formula [9]:

$$\psi_s(m_\tau)=\frac{1}{N-m_\tau}\sum_{k=1}^{N-m_\tau}\begin{bmatrix}V_S(k)V_S(k+m_\tau)+\\ V_R(k)V_S(k+m_\tau)+V_S(k)V_R(k+m_\tau)\end{bmatrix},$$
$$(m_\tau=0,1,\ldots,M-1) \qquad (3.16)$$

In the low-frequency limit $2\pi fT_0<<1$ for the singular component $S_s(f)$ is useful interpolation formula [9]

$$S_s(f)\approx\frac{S_s(0)}{1+(2\pi fT_0)^{n_0}} \qquad (3.17)$$

As approximate value of parameter $S_s(0)$ in the low-frequency area of a function graph (or) preceding a frequency interval in which there are considerable changes of this dependence in double logarithmic scale some value $S^*(0)$ which is given sense of parameter $S_s(0)$ in interpolation expression (3.17) [26]. We believe that in $S_s(f)<S_s^*(0)$ the area $2\pi fT_0<1$.

The parameters T_0 and n_0 the interpolation relationship (3.17) for occasional bursts contribution to the $S(f)$ (or $|S(f)|$)) is defined by the matching condition (using the least squares method) range $S(f)$ on the right side of (3.17). To do this, we use the following algorithm [27].

Algorithm 3.1.

We introduce, $S_S^*(0)$, $RSS^*=10^{10}$, $\lambda=0$.

1. Setting $S_s(0):=S_s^*(0)$ andassessment T_0 and

Consider the regression

$$y=ax+b, \qquad (3.18)$$

where $y=ln\left|\frac{S_S(0)}{S_S(f)}-1\right|$, $x=ln2\pi f$, $a=n_0$, $b=n_0lnT_0$

According to the OLS $\hat{a},\hat{b}$ estimate, we find the valuation $\hat{n}_0=\hat{a}$, $\hat{T}_0=\exp\left(\frac{\hat{b}}{\hat{a}}\right)$.

2. We calculate the regression (3.18) is the sum of squared residuals (residual squares sum - RSS):

$$RSS^{(1)}=\sum_{m=0}^{M-1}\left[y_m-\left(\hat{a}x_m+\hat{b}\right)\right]^2$$

Where y_m and x_m appropriate frequencies $f_m=\frac{m}{M\cdot\Delta t}$

if $RSS^{(1)}<RSS^*$, то $RSS^*:=RSS^{(1)}$, $n_0^*:=\hat{n}_0$, $T_0^*:=\hat{T}_0$.

3. shallow estimate $S_s(0):=S_s^*(0)$, $n_0^*:=\hat{n}_0$, estimate T_0.

Consider the regression

$$y=ax+b \quad (b=0) \qquad (3.19)$$

Where $y=\left|\frac{S_S(0)}{S_S(f)}-1\right|^{1/n_0}$, $x=2\pi f$, $a=T_0$

by OLS $\hat{a}$ estimate is appreciated $\hat{T}_0=\hat{a}$.

4. We calculate the regression (3.19)

$$RSS^{(2)}=\sum_{m=0}^{M-1}\left[y_m-\hat{a}x_m\right]^2$$

If $RSS^{(2)}<RSS^*$, in fact $RSS^*:=RSS^{(2)}$, $T_0^*:=T_0$.

5. Believing, $S_s(0):=S_s^*(0)$, $T_0:=T_0^*$, we estimate n_0.

Consider the regression

$$y=ax+b \quad (b=0), \qquad (3.20)$$

where

$$y=ln\left|\frac{S_S(0)}{S_S(f)}-1\right|,\ x=ln(2\pi fT_0),\ a=n_0$$

by OLS $\hat{a}$ -estimate is appreciated $\hat{n}_0=\hat{a}$.

6. We calculate the regression (3.20)

$$RSS^{(3)}=\sum_{m=0}^{M-1}\left[y_m-\hat{a}x_m\right]^2$$

If $RSS^{(3)}<RSS^*$, in fact $RSS^*:=RSS^{(3)}$, $n_0:=\hat{n}_0$.

7. Assuming $T_0:=T_0^*$, $n_0:=\hat{n}_0$ estimate $S_S(0)$.

Consider the regression

$$y=ax+b \quad (b=0), \qquad (3.21)$$

where $y=S_s(f)$, $x=\frac{1}{1+(2\pi fT_0)^{n_0}}$, $a=S_S(0)$.

According to the OLS $\hat{a}$ estimate is appreciated $S_S(0)=\hat{a}$.

8. Calculate the regression (3.21)

$$RSS^{(4)} = \sum_{m=0}^{M-1} \left[y_m - \hat{a} x_m \right]^2$$

If $\left(RSS^{(4)} < RSS^*\right) \wedge \left(\hat{S}_s(0) \geq S_s^*(0)\right)$, то $RSS^* := RSS^{(4)}$, $S_s^*(0) = \hat{\hat{S}}_s(0)$, $\lambda := \lambda + 1$.

9. If $\lambda < 10$, then go to step 1.

10. Print $S_S^*(0)$, T_0^*, n_0^*, RSS^*

11. END.

Thus, formula (3.17) can be written as

$$S_s(f) \approx \frac{S_s(0)}{1 + \left(2\pi f T_o^*\right)^{n_o^*}}. \tag{3.22}$$

Applying the algorithm 3.1 the scalar dynamic signal $V(t) = x(t)$, where $x(t)$ - the solution of equation (2.1), you can take the value T_0^* of the time of reinjection, since the main contribution to the yields $S(f)$ are usually singular component with $S_s(f)$ a small amplitude low-frequency component of the signal $V_R(t)$.

4. IDENTIFICATION OF THE OSCILLATOR PARAMETERS OF THE VAN DER POL OSCILLATOR USING WAVELET ANALYSIS

Any function $f(t)$ square integrable $R = (-\infty, \infty)$ in the space can be expanded at a given level of resolution $j = j_0$ in the wavelet series [28,29]

$$f(t) = \sum_k s_{j_0,k}\, \phi_{j_0,k}(t) + \sum_{j \geq j_0} \sum_k d_{j,k}\, \psi_{j,k}(t), \tag{4.1}$$

Where

$$\phi_{j,k}(t) = 2^{j/2} \phi\left(2^j t - k\right), \quad \psi_{j,k}(t) = 2^{j/2} \psi\left(2^j t - k\right). \tag{4.2}$$

As introduced, $\phi(t)$ and $\psi(t)$ we will take Daubechies [30] scaling (scale) $\phi_M(t)$ wavelet basis function ${}_M\psi(t)$ and defined by the equations:

$$\phi(t) = \sqrt{2} \cdot \sum_{k=0}^{2M-1} h_k \phi(2t - k), \quad \psi(t) = \sqrt{2} \cdot \sum_{k=0}^{2M-1} g_k \phi(2t - k), \tag{4.3}$$

Where $g_k = (-1)^k \cdot h_{2M-k-1}$, M- positive integer.

Within the framework of multiresolution analysis (MRA) [31] functions $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ serve as a high-frequency and low-frequency filters, RHR, respectively h_k. The general properties of the scaling functions and wavelet coefficients are uniquely determined. Example of calculation for Daubechies $\left(D^4\right)$ wavelet is given in [28].

For Daubechies h_k wavelets are real and

$$s_{j,k} = \left\langle f, \phi_{j,k} \right\rangle, \quad d_{j,k} = \left\langle f, \psi_{j,k} \right\rangle, \tag{4.4}$$

where for any $f_1(t)$ and $f_2(t)$ ($L_2(R)$ ($L_2(R)$- space of square-integrable functions on R

$$\left\langle f_1, f_2 \right\rangle = \int_R f_1(t) f_2(t)\, dt. \tag{4.5}$$

Using formulas fast wavelet transform (FWT)

$$s_{j,n} = \sum_{k=0}^{2M-1} h_k \cdot s_{j-1,2n+k}, \quad d_{j,n} = \sum_{k=0}^{2M-1} g_k \cdot s_{j-1,2n+k}. \tag{4.6}$$

When time signal $f \in L_2(R)$ analysis should choose the thinnest scale to the subsequent signal synthesis receive it in its original form. We can assume that such a scale is associated with the level of permissions $j = 0$. Therefore, the analysis begins with the calculation

$$s_{0,k} := \left\langle f, \phi_{o,k} \right\rangle = \int_R f(t) \phi(t - k)\, dt. \tag{4.7}$$

These values can be calculated, for example by means of numerical integration. In the case where f the initially set as a discrete array $\{f(k)\}$, $k \in Z$ (Z - set of integers), just suppose

$$s_{0,k} := f(k), \quad k \in Z. \tag{4.8}$$

Applying FWT (4.7), we can now calculate all the factors $s_{j,n}, d_{j,n}$.

Let's consider action of the operator $T^{(n)} = \frac{d^n}{dt^n}$ $(n \geq 1)$ on function $f(t)$, $t \in R$, within MRA in $L_2(R)$. Let's designate matrix elements of this operator through.

$T_{SS}^{(n)}(j,k;\, j,k')$, $T_{SD}^{(n)}(j,k;\, j,k')$, $T_{DS}^{(n)}(j,k;\, j,k')$, $T_{DD}^{(n)}(j,k;\, j,k')$. Here

$$T_{SS}^{(n)}\left(j,k;\ j,k'\right)=\int_R \phi_{j,k}\left(t\right)\left(T^{(n)}\phi_{j,k'}\left(t\right)\right)dt, \tag{4.9}$$

replacement of the lower indices $S\to D\left(D\to S\right)$ in the left side of (4.9) corresponds to the substitution $\phi\to\psi\left(\psi\to\phi\right)$ of the integral sign on the right side. We denote further $r_{j,l;j,l'}^{(n)}=T_{SS}^{(n)}\left(j,l;j,l'\right)$, $r_k^{(n)}=T_{SS}^{(n)}\left(0,0;0,k\right)$. Obviously, where $r_{j,k}^{(k)}=r_{j,0;j,k}^{(k)}$.

Elements $r_{j,k}^{(n)}$ of the same level are related

$$r_{j,k}^{(n)}=2\sum_{i=0}^{L}\sum_{m=0}^{L}h_i\cdot h_m\cdot r_{j,2k-i+m}^{(n)},\quad \left(L=2M-1\right), \tag{4.10}$$

and the elements adjacent levels – ratio

$$r_{j+1,l;j+1,l'}^{(n)}=2^n\cdot r_{j,l;j,l'}^{(n)}. \tag{4.11}$$

Conditions normalization coefficients $r_{0,k}^{(n)}$ is determined as [28]

$$\sum_{k=1}^{L}k\cdot r_{0,k}^{(n)}=n!. \tag{4.12}$$

If $j=0$ (4.10) for the value of $r_{0,k}^{(n)}$ (the $r_k^{(n)}$) we obtain the system of equations

$$r_k^{(n)}=2\sum_{i=0}^{L}\sum_{m=0}^{L}h_i\cdot h_m\cdot r_{2k-i+m}^{(n)},\quad \left(k=0,1,\ldots,L-2\right). \tag{4.13}$$

In the domain of the L wavelet coefficients $r_k^{(n)}$ in length have the symmetry property

$$r_k^{(n)}=\left(-1\right)^k\cdot r_{-k}^{(n)}\quad \left(k=1,2,\ldots\right). \tag{4.14}$$

Given a $L'\geq n$ $\left(L'=L-1\right)$ system of linear algebraic equations (LAE) (4.12-4.14) has a unique exact solution $u=\left(r_0^{(n)},r_1^{(n)},\ldots,r_{L-2}^{(n)}\right)$. The matrix elements of the differentiation operator recurrently expressed in terms of matrix elements of the operator of differentiation $T^{(n-1)}=\dfrac{d^{n-1}}{dt^{n-1}}$ [32], when they are given below**.**

Solving Systems (4.12) - (4.14) $n=1$ for all matrix elements $r_{0,l;0,l'}^{(k)}=r_k^{(1)}\big|_{k=l'-l}$ can be recovered from the elements. $T_{SS}^{(1)}\left(j,l;\ j,l'\right)=r_{j,l;j,l'}^{(1)}\left(j\geq 1\right)$ Using the recurrence relation (4.11). The remaining matrix elements are $n=1$ defined as follows. By implementing the equivalent of (4.10) in the $n=1$ expression

$$T_{SS}^{(1)}\left(j,l;\ j,l'\right)=2\sum_{i=0}^{L}\sum_{m=0}^{L}h_i\cdot h_m\cdot r_{j,2l+i;j,2l'+m} \tag{4.15}$$

replacement indexes $S\to D$ and respective substitute $h\to g$, obtain for all $j\geq 0$ presentation

$$\begin{aligned}
T_{SD}^{(1)}\left(j,l;\ j,l'\right)&=2\sum_{i=0}^{L}\sum_{m=0}^{L}h_i\cdot g_m\cdot r_{j,2l+i;j,2l'+m}^{(1)},\\
T_{DS}^{(1)}\left(j,l;\ j,l'\right)&=2\sum_{i=0}^{L}\sum_{m=0}^{L}g_i\cdot h_m\cdot r_{j,2l+i;j,2l'+m}^{(1)},\\
T_{DD}^{(1)}\left(j,l;\ j,l'\right)&=2\sum_{i=0}^{L}\sum_{m=0}^{L}g_i\cdot g_m\cdot r_{j,2l+i;j,2l'+m}^{(1)}.
\end{aligned} \tag{4.16}$$

Let $f\left(t\right)$ represented as $g\left(t\right)=T^{(n)}f\left(t\right)$ a series of wavelet coefficients $\left({}_f s_{j,k};\ {}_f d_{j,k}\right)$ and $\left({}_g s_{j,k};\ {}_g d_{j,k}\right)$. Then

$$\begin{aligned}
{}_g s_{j,k}&=\sum_{k'=0}^{L}\left(T_{SS}^{(n)}\left(j,k;j,k'\right)\cdot {}_f s_{j,k'}+T_{SD}^{(n)}\left(j,k;j,k'\right)\cdot {}_f d_{j,k'}\right),\\
{}_g d_{j,k}&=\sum_{k'=0}^{L}\left(T_{DS}^{(n)}\left(j,k;j,k'\right)\cdot {}_f s_{j,k'}+T_{DD}^{(n)}\left(j,k;j,k'\right)\cdot {}_f d_{j,k'}\right).
\end{aligned} \tag{4.17}$$

The matrix elements are $n=2$ connected to the matrix elements $n=1$ by relations with

$$\begin{aligned}
T_{SS}^{(2)}\left(j,k;j,k_2\right)&=\sum_{k_1=0}^{L}\begin{pmatrix}T_{SS}^{(1)}\left(j,k;j,k_1\right)\cdot T_{SS}^{(1)}\left(j,k;j,k_2\right)\\+T_{SS}^{(1)}\left(j,k;j,k_1\right)\cdot T_{DS}^{(1)}\left(j,k;j,k_2\right)\end{pmatrix},\\
T_{SD}^{(2)}\left(j,k;j,k_2\right)&=\sum_{k'=0}^{L}\begin{pmatrix}T_{SS}^{(1)}\left(j,k;j,k_1\right)\cdot T_{SD}^{(1)}\left(j,k;j,k_2\right)\\+T_{SD}^{(1)}\left(j,k;j,k_1\right)\cdot T_{DD}^{(1)}\left(j,k;j,k_2\right)\end{pmatrix},\\
T_{DS}^{(2)}\left(j,k;j,k_2\right)&=\sum_{k'=0}^{L}\begin{pmatrix}T_{DS}^{(1)}\left(j,k;j,k_1\right)\cdot T_{SS}^{(1)}\left(j,k;j,k_2\right)\\+T_{DD}^{(1)}\left(j,k;j,k_1\right)\cdot T_{DS}^{(1)}\left(j,k;j,k_2\right)\end{pmatrix},\\
T_{DD}^{(2)}\left(j,k;j,k_2\right)&=\sum_{k'=0}^{L}\begin{pmatrix}T_{DS}^{(1)}\left(j,k;j,k_1\right)\cdot T_{SD}^{(1)}\left(j,k;j,k_2\right)\\+T_{DD}^{(1)}\left(j,k;j,k_1\right)\cdot T_{DD}^{(1)}\left(j,k;j,k_2\right)\end{pmatrix}.
\end{aligned} \tag{4.18}$$

Replacing in the equation (1.11) oscillator Van der Pol $x\left(t\right), dx/dt$ and d^2x/dt^2 expansions in the wavelet basis with the carrier. Applying the formula of differentiation of (4.12) and taking t the points $k=0,1,\ldots,L$, we get in the parameter λ type LAE

$$A\Lambda=b \tag{4.19}$$

where $A=\left(a_0,a_1,\ldots,a_L\right)^T$ - the vector of one element λ, $b=\left(b_0,b_1,\ldots,b_L\right)^T$ T - a sign of transposition.

Overdetermined system (4.19) does not have an exact solution, so instead of an exact solution is organized search of the vector Λ that will best meet all the equations of the system (4.19), ie, minimize the residual (the difference between the vector $A\Lambda$ and the vector), ie $|A\Lambda - b| \to \min$. Such a solution can be obtained by Isolve function MATHCAD system.

hus, the calculation of the original signal wavelet coefficients using fast wavelet transform and application of the formulas of differentiation of the discrete wavelet decomposition reduces the problem of determining the oscillator parameters directly to solving a system of linear algebraic equations directly in the space of wavelet coefficients, which significantly increases the speed of calculations in comparison with method of direct and inverse wavelet transforms. This approach has undoubted advantages compared with the numerical $x(t)$ signal by differentiating the approximate mathematical formulas. In addition, if the signal to be analyzed is added to normally distributed random process, the formal application of numerical formulas differen¬tsirovaniya based on Newton interpolation polynomials leads to large errors estimates for the derivatives, as shown in [16]. based wavelet approach provides at least an order of magnitude less than the magnitude of error, and this error can be reduced even further by a suitable choice of wavelet basis. The apparent advantage of wavelet perobrazovaniya is the ability to eliminate the incorrect operation of the numerical differentiation of noisy time series by moving into the space of wavelet coefficients

5. SYNCHRONIZATION OF THE NOISE OF AUTO-OSCILLATIONS IN THE OSCILLATORY PATTERN OF THE AVERAGED HEART RATE UNDER THE INFLUENCE OF EXTERNAL PERIODIC

Example of synchronization of self-oscillations of the noise of nonlinear oscillator under the influence of periodic external force is described in [33]. In this paper we investigated the capture of the human heart rhythm weak periodic signal. During the experiment, the subjects were at rest in front of the monitor screen while the computer periodically generate acoustic and visual cues - both with sound pulses appear on the screen of colored squares.

The response to the impact determined by ECG, a release in a standard way. It is well known that every normal cardiac cycle contains a very sharp, well-isolated peak called R- wave (Figure **7** [10]). the spacing between neighboring R- usually peaks is taken as the interval between heartbeats

Naturally, the contraction of the heart - not the periodic self-oscillation, and is easy to see that the intervals between the beats change significantly over time; This variability is well known in physiology. Therefore, we can only talk about the middle period and heart rate. To estimate the average frequency is enough to count the number n of off R-peak for a certain period of time τ and $\bar{f} = n/\tau$ find.

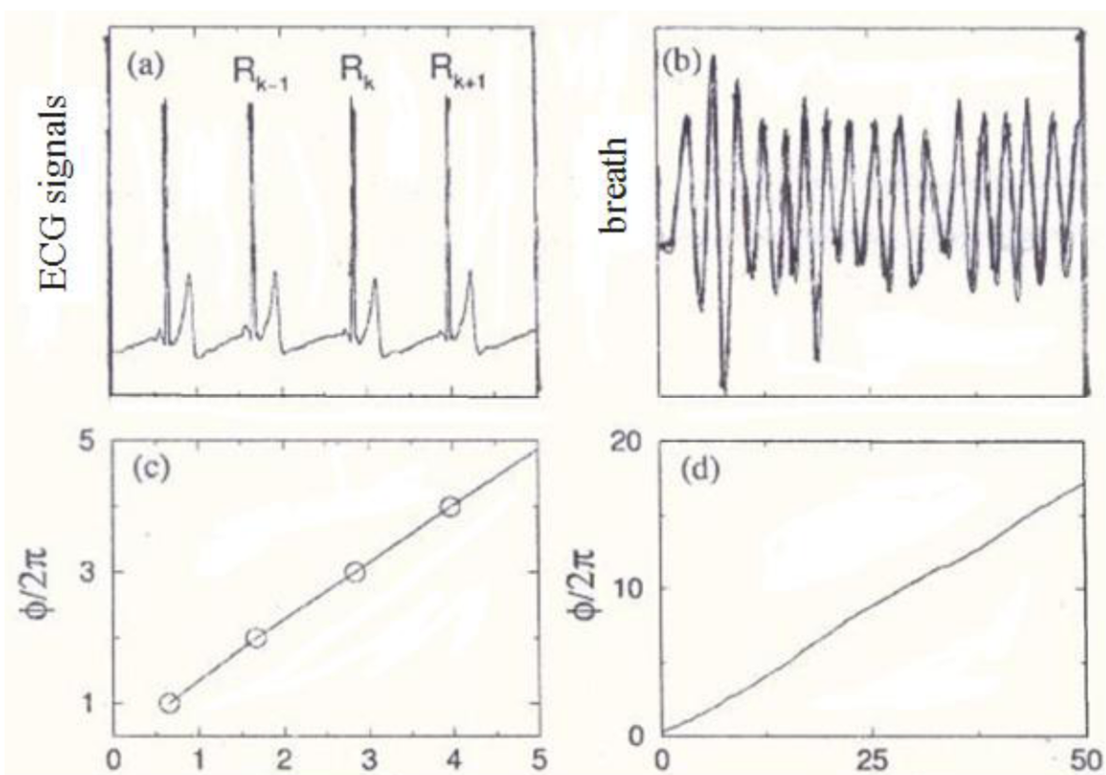


Figure 7: A short segment of the electrocardiogram (ECG) labeled with R-teeth (**a**) recording the breath and segment (**b**); both signals are in arbitrary units. Phase ECG is a piecewise linear function of time; since the advent of R-wave circles marked (**c**). Respiratory phase obtained through a Hilbert transform (**d**).

R-wave sequence can be viewed as a series of point events occurring at a time t_k, $k = 1,2,\dots$. The phase of this process can be easily calculated. Indeed, the time interval between two R - wave corresponds to one of the cardiac cycle when a cycle to understand the interval between two nearly identical states 2π of the system. Consequently, the phase increases during this interval exactly. Thus, the t_k times may be attributed to phase values $\phi(t_k) = 2\pi k$ and random points in time $t_k < t < t_{k+1}$ to determine the phase by a linear interpolation between these values:

$$\phi(t) = 2\pi k + 2\pi \frac{t - t_k}{t_{k+1} - t_k} \quad (5.1)$$

As described in [10] experiments, the frequency of the test rate f_0, without affecting initially determined. Then subjected to external stimulation test with different f frequencies $(0,75-1,25)f_0$ from one test to another. The resulting curves the difference $(F - f_0)/f_0$ in the observed frequency f/f_0 of the detuning is shown in Figure **8** [10].

Shelf near $f \approx f_0$ Figure **8**, although not perfectly horizontal, it indicates the seizure frequency.

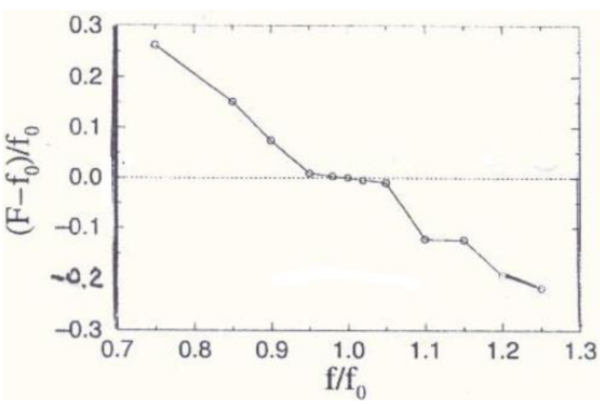


Figure 8: The observed average frequency F heart rate as a function of frequency f weak external stimulus. This experimental curve corresponds to the curve of the frequency difference of detuning for the noise of oscillation with the external force.

Type I intermittency phenomenon investigated for RR-intervals in previous studies [34,35]. The transition from the regular state of the process to chaos through intermittency of type I is characterized by the manifestation of the phenomenon of self-organized criticality (SOC), or, as they say, weak chaos, leading to system instability and flicker noise with power spectrum RR-intervals nature of power. This behavior - intervalogrammy is a diagnostic feature that allows to distinguish the main syndromes of the cardiovascular system.

REFERANCE

[1] Schuster G. Determinate chaos. Introduction /Trans. with Eng - M Mir 1988; 240.

[2] Malinetsky GG, Potapov AB. Nonlinear dynamics and chaos. Basic concepts. Publishing house of the 2nd. M.: KomKniga 2000; 240.

[3] Dubois M, Rubio M, Berge P. Experimental evidence of intermiasttencies associated with a subharmonic bifurcation. Phys Rev Lett 1983; 51: 1446. https://doi.org/10.1103/PhysRevLett.51.1446

[4] Platt N, Spiegel EA, Fresser C. On-off intermittency: a mechanism for bursting. Phys Rev Lett 1993; 70(3): 279. https://doi.org/10.1103/PhysRevLett.70.279

[5] Heagy JF, Platt N, Hammel SM. Characterization of on-off intermittency. Phys Rev E 1994; 49(2): 1140. https://doi.org/10.1103/PhysRevE.49.1140

[6] Pikovsky AS, Osipov GV, Rosenblum MG, Zacs M, Kurths JU. Attractor-repeller collision and eyelet intermittency at the transition to phase synchronization. Phys Rev Lett 1997; 79(1): 47. https://doi.org/10.1103/PhysRevLett.79.47

[7] Lee KJ, Kwak Y, Lim TK. Phase jams near a phase synchronization transition in systems of two coupled chaotic oscillators. Phys Rev Lett 1998; 81(2): 321. https://doi.org/10.1103/PhysRevLett.81.321

[8] Hramov AE, Koronovsky AA, Kurovskaya MK, Boccaletti S. Ring intermittency in coupled chaotic oscillators at the boundary of phase synchronization. Phys Rev Lett 2006; 97: 114107. https://doi.org/10.1103/PhysRevLett.97.114101

[9] Timashev SF. Flicker - noise spectroscopy: The information in chaotic signals. M FIZMATLIT 2007; 248.

[10] Pikovsky AS, Rosenblum MG, Kurths JU. Synchronization: a universal concept in nonlinear sciences, Cambridge University Press, 2001; p. 433.

[11] Kuznetsov AP, Kuznetsov SP, Ryskin NM. Nonlinear vibrations. M FIZMATLIT 2002; 292.

[12] Landa PS. Nonlinear Waves. M LIBROKOM 2010; 552.

[13] Pikovsky AS, Rosenblum MG, Kurths JU. Phase synchronization in regular and chaotic systems. Int J Bifurcation and Chaos 2000; 10(10): 2291. https://doi.org/10.1142/S0218127400001481

[14] Khramov AE, Koronovsky AA, Kurovskaya MK. Two types of phase synchronization systems. Phys Rev E 2007; 3: 036,205.

[15] Berge P, Pomeau Y, Vidal Ch. L'ordre dans le chaos. Hermann, Paris, 1988; p. 351.

[16] Koronovsky AA, Kurovskaya MC, Khramov AE. Distribution of the laminar phases for the intermittency of type I in the presence of noise. Math universities "Applied Nonlinear Dynamics" 2009; 17(5): 43-59.

[17] Anischenko VS, Vadivasova TE, Astakhov VV. Nonlinear dynamics of chaotic and stochastic systems. Fundamentals and Selected Problems / Ed. V.S. Anischenko. - Saratov: Izd Sarat. University Press, 1999; p. 368.

[18] Hirsch JE, Huberman BA, Scalapino DJ. Theory of intermittency. Phys Rev A 1982; 25(1): 519. https://doi.org/10.1103/PhysRevA.25.519

[19] Hirsch JE, Nauenberg M, Scalapino DJ. Phys Lett A 1982; 87: 391. https://doi.org/10.1016/0375-9601(82)90165-7

[20] Crutchfield JP, Farmer JD, Huberman BA. Fluctuation and simple chaotic dynamics. Physic Reports (Review Section of Physics Letters) 1982; 92(2): 45-82. https://doi.org/10.1016/0370-1573(82)90089-8

[21] Kye W-H, Kim C-M. Characteristics relations of type-I intermittency in the presence of noise. Phys Rev E 2000; 62(5): 6304. https://doi.org/10.1103/PhysRevE.62.6304

[22] Jin-Hang C, Myung-Suk K, Young-Jai P, Kim C-M. Experimental observation of the characteristic relations of type-I intermittency in the presence of noise. Phys Rev E 2002; 65(3): 036,222.

[23] Brillindzher D. Time series. Processing of data and theory. M Mir 1980; p. 536.

[24] Timashev SF, Vstovskiy GV. Flicker - noise spectroscopy in the analysis of chaotic time series of dynamic variables and the problem of relations "signal - noise". Electrochemistry 2003; 39: 156-169.

[25] Samarsky AA, Gulin AV. Stability of difference schemes. M Nauka 1973; 415.

[26] Vstovsky GV. Elements of Information Physics - M.: Moscow State Pedagogical University, 2002; 258.

[27] Dzhanahmedov AH, Dyshin OA, Javadov MJ. Synergetics and Fractals in tribology. - Baku: APOSTROFF 2014; 504.

[28] Dremin IM, Ivanov OV, Nechitailo VA. Wavelets and their uses. Successes of physical sciences 2001; 171(5): 465-501.

[29] Blatter K. Wavelet analysis. Basic theory. / Transl. from English. - M .: Technosphere 2006; 272.

[30] Daubechies I. wavelet Ten lectures. Moscow Izhevsk: IKI, 2004; 163.

[31] Mallat S. Multiresolution approximation and wavelets. Trans Amer Math Soc 1989; 315: 69-88.

[32] Dyshin OA. The method of calculating the matrix of sustainable performance in wavelet basis functions of differential operators. Proceedings of the Azerbaijan. Techn. University, t. VII (27), №3, 2008, p. 76-82.

[33] Anishchenko VS, Balanov AG, Janson NB, Igosheva NB, Bordyungov GV. Entrainment between heart rate and weak noninvasive forcing. Int J Bifurc Chaos 2000; 10: 2339-2348. https://doi.org/10.1142/S0218127400001468

[34] Abdullayev NT, Dyshin OA, Dzhabieva AD. Analysis of the instability of the time series of RR in terms of criticality samoorganizovannoy. Abstracts of the International scientific-practical conference "Information technologies and computer engineering", Vinnitsa, Ukraine on May 19-21, 2010; pp. 464-465.

[35] Abdullayev NT, Dyshin OA, Abbaskuliev AS. Evaluation of instability -intervalogramm. Medical Technique 2011; №3 (267): 34-37.

Received on 20-10-2016 Accepted on 11-11-2016 Published on 18-11-2016

DOI: http://dx.doi.org/10.12970/2311-052X.2016.04.02.2